Studying the effects of minimal length in large extra dimensional models in the jet + missing energy channels at hadron colliders

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Abstract. Theories of quantum gravity suggest the existence of a minimal length scale. We study the consequences of a particular implementation of the idea of a minimal length scale in the model of large extra dimensions, the ADD model. To do this we have looked at real graviton production in association with a jet at hadron colliders. In the minimal length scenario, the bounds on the effective string scale are significantly less stringent than those derived in the conventional Arkani-Hamed–Dimopoulos–Dvali model, both at the upgraded Tevatron and at the Large Hadron Collider.

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1 Introduction

There are reasons to believe that at the Planck scale, the scale at which gravity becomes a quantum phenomenon, the very structure of space-time may change. That this may happen is suggested even by general relativity. A quantum mechanical particle of momentum p in the presence of a classical gravitational field (the latter described by Einstein's equations) causes the metric g to fluctuate. This induces an additional uncertainty in position, given by $l_{\mathrm{Pl}}^2 \Delta p$, where l_{Pl} is the Planck length. Thus the usual quantum mechanical uncertainty relation gets modified to

$$\Delta x \gtrsim \frac{1}{\Delta p} + l_{\mathrm{Pl}}^2 \Delta p \,. \tag{1}$$

At high energies, the second term can become significant and may lead to important deviations from the usual quantum mechanics. For example, in the usual quantum mechanics, Δx is large at low values of the momenta but can become small at high momenta which can provide a higher resolution. With the modified uncertainty relation even at high momenta Δx is limited in resolution because of strong curvature effects. In other words, independent of momentum, Δx is always larger than a minimal length scale l_{Pl}. The appearance of the minimal length in the classical theory of gravity should tell us that it is no surprise to expect that such a conclusion becomes even more inevitable in a quantum theory of gravity. Indeed, a whole range of quantum gravity models predict the existence of a minimal length [1, 2]. However, because the Planck length is so

minuscule, it is of consequence only to the physics of the early universe, if at all [3]. There is a twist in the plot, however, and that is due to the development of brane-world physics. This new paradigm in physics has far-reaching implications, but of our immediate interest is the fact that there are brane-world models which allow for a realization of quantum gravity effects at very low energy scales – as low as a TeV. In this paper, we will deal with the simplest of such realizations, which is a model due to Arkani-Hamed, Dimopoulos and Dvali.

In the model of Arkani-Hamed, Dimopoulos and Dvali, the so-called ADD model [4], extra space dimensions are compactified to have large volume moduli, and thereby it becomes possible to lower the scale of quantum gravity from the Planck scale to the TeV scale. In the ADD model, the gauge interactions are confined to a 3-brane while gravity propagates in all the available dimensions. The higher dimensional Planck scale M_{S} in the ADD model is related to the usual Planck scale by $M_{\mathrm{Pl}}^2 = R^d M_{\mathrm{S}}^{d+2}$, where R is the radius of compactification and d is the number of extra space dimensions. For appropriate choices of R and d, the fundamental (higher dimensional) Planck or string scale can be brought down to $M_{\mathrm{S}} \sim 1\,\mathrm{TeV}$. This scale can be probed at present and future colliders. Various signals of graviton production and virtual graviton exchange have been studied [5, 6] and reviewed [7, 8] in recent years. In the context of the ADD model with $M_{\mathrm{S}} \sim 1\,\mathrm{TeV}$, the minimal length hypothesis is phenomenologically interesting if we take it to be around an inverse TeV, viz. $l_{\mathrm{Pl}} \sim 1/M_{\mathrm{S}}$. For a review, see [9].

Different applications of the minimal length scenario (MLS) have been discussed in [10–12]. In an earlier analysis [12], we have studied how the MLS hypothesis influ-

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ences the dilepton production process in hadron colliders. In the ADD model, for such processes involving virtual gravitons one has to sum over an infinite tower of graviton Kaluza–Klein (KK) states at the amplitude level. The result is divergent and to cure the divergence an ad-hoc cutoff of the order of M_S is used. However, in the MLS scenario, the minimal length acts as an ultraviolet regulator and allows one to sum over the entire KK graviton tower by smoothly cutting off the contribution of higher energy KK states rendering the amplitude finite. Another important modification comes from the rescaling of the momentum measure leading to an alteration in the phase space integration. These, as we observed in [12], lead to a significant deviation of the bound on M_S from the one obtained in the conventional ADD picture without the MLS hypothesis.

In the present analysis, we consider processes involving the emission of real gravitons in hadron colliders, namely, the upgraded Tevatron and the future Large Hadron Collider (LHC). The partonic sub-processes are $P_1 P_2 \to P_3 G_{\mathbf{n}}$, where P_i are the appropriate quarks/gluons. Here $G_{\mathbf{n}}$ means a real graviton with KK index $\mathbf{n} = (n_1, n_2, \dots, n_d)$, where d is the number of extra dimensions. For a given process, one must sum over such external modes at the cross section level. This KK summation gets modified in the MLS scenario. The phase space integration also gets modified in the MLS scenario. Again, as we shall see towards the end, the bounds that one obtains on M_S in the conventional ADD model from the jet plus missing energy mode in the hadron colliders are considerably weakened if we implement the MLS hypothesis.

2 The MLS scenario and the jet + missing energy channel

Specifically, by the MLS scenario we mean the ADD model with a minimal length l_{Pl} of the order of TeV^{-1} incorporated in it. Since the uncertainty in a position measurement now cannot be smaller than l_{Pl}, now the standard commutation relation between position and momentum has to be modified [13]. In the MLS scheme, the Compton wavelength ($\lambda = 2\pi/k$) of a particle cannot be arbitrarily small. We suppose that even though the wave vector k is bounded from above, the momentum p can be as large as possible. In the same way, the frequency ω is restricted from above, while the energy E can grow arbitrarily. This means that the standard relations $p = \hbar k$ and $E = \hbar \omega$ need to be modified. This can be realized by introducing the Unruh ansatz [14]

$$l_{\mathrm{Pl}} k(p) = \tanh^{1/\gamma}\left[\left(\frac{p}{M_S}\right)^\gamma\right],$$
$$l_{\mathrm{Pl}} \omega(E) = \tanh^{1/\gamma}\left[\left(\frac{E}{M_S}\right)^\gamma\right], \quad (2)$$

where γ is a positive constant. Equation (2) captures the essence of a minimal length scale: as p (or E) becomes very large, k (or ω) approaches the upper bound $1/l_{\mathrm{Pl}}$, and

hence one cannot probe arbitrarily small length and time scales. The generalized position–momentum and energy–time uncertainty principle can now be written in a Lorentz covariant form as

$$[x^\nu, p_\mu] = \mathrm{i}\frac{\partial p_\mu}{\partial k_\nu}. \quad (3)$$

As explained in [10, 13], the effect of (2) can conveniently be accounted for by a simple redefinition of the momentum measure, which in one dimension is given by

$$\mathrm{d}p \longrightarrow \mathrm{d}p\frac{\partial k}{\partial p}. \quad (4)$$

The Lorentz invariant 4-momentum measure is modified as

$$\mathrm{d}^4 p \to \mathrm{d}^4 p \det\left(\frac{\partial k_\mu}{\partial p_\nu}\right) = \mathrm{d}^4 p \prod_\nu \frac{\partial k_\nu}{\partial p_\nu}, \quad (5)$$

where the Jacobian matrix $\frac{\partial k_\nu}{\partial p_\nu}$ can be kept diagonal.

In this paper we study how processes involving real graviton emissions in hadron colliders are going to be influenced by the MLS hypothesis. The emitted gravitons will disappear carrying missing energy, so the events that we look for are jets plus missing energy. The partonic sub-processes that contribute to this channel are $q\bar{q} \to gG_{\mathbf{n}}$, $qg \to qG_{\mathbf{n}}$, $\bar{q}g \to \bar{q}G_{\mathbf{n}}$, and $gg \to gG_{\mathbf{n}}$. The differential cross section for a particular sub-process (labelled i) for a given KK graviton mode has the form

$$\frac{\mathrm{d}\sigma_i^{\mathbf{n}}}{\mathrm{d}\cos\theta} = G_{\mathrm{N}} f_i\left(m_{\mathbf{n}}^2, s, t, u\right), \quad (6)$$

where $m_{\mathbf{n}}$ is the mass of $G_{\mathbf{n}}$, and the f_i are functions of the Mandelstam variables s, t, u whose explicit expressions are given in [6, 15]. These cross sections have to be convoluted with the density of KK states given by [5]

$$\rho(m_{\mathbf{n}}) = \frac{R^d m_{\mathbf{n}}^{d-2}}{(4\pi)^{d/2}\Gamma(d/2)}. \quad (7)$$

Noting that one can express $G_{\mathrm{N}} = (4\pi)^{d/2}\Gamma(d/2)/(2M_S^{d+2}R^d)$, the differential cross section summed over the kinematically allowed KK graviton tower is given by

$$\frac{\mathrm{d}\sigma_i^{\mathrm{ADD}}}{\mathrm{d}\cos\theta} = \frac{1}{2M_S^4}\int \mathrm{d}m_{\mathbf{n}}^2 \left(\frac{m_{\mathbf{n}}^2}{M_S^2}\right)^{\frac{d-2}{2}} f_i(m_{\mathbf{n}}^2, s, t, u), \quad (8)$$

where the lower limit of integration is zero and the upper limit corresponds to the maximum allowed graviton energy as $\sqrt{s}/2$. An important thing that has to be noticed here is that while the usual 4-dimensional graviton emission cross section goes like $1/M_{\mathrm{Pl}}^4$, the effect of KK summation jacks up the contribution which goes effectively as $1/M_S^4$. This is the ADD effect. Now, how does the MLS hypothesis modify the KK summation? For analytic simplicity we take $\gamma = 1$ and set $l_{\mathrm{Pl}} M_S = \hbar = 1$. The departure from these assumptions will be discussed later. The Unruh relations (2)

will modulate the integral in (8) by a factor $\partial\omega/\partial E$. The MLS-influenced differential cross section is given by

$$\frac{\mathrm{d}\sigma_i^{\mathrm{MLS}}}{\mathrm{d}\cos\theta} = \frac{1}{2M_{\mathrm{S}}^4} \int \mathrm{d}\, m_{\mathbf{n}}^2 \left(\frac{m_{\mathbf{n}}^2}{M_{\mathrm{S}}^2}\right)^{\frac{d-2}{2}}$$
$$\times \operatorname{sech}^2\left(\frac{m_{\mathbf{n}}}{M_{\mathrm{S}}}\right) f_i\left(m_{\mathbf{n}}^2, s, t, u\right). \qquad (9)$$

For large argument, the sech-square function falls exponentially; thus the contributions from high mass KK states will be smoothly cut off.

Another important modification due to the MLS is the change in the cross section due to the rescaling of the momentum measure given in (5). It is straightforward to check, as derived in [10], that the phase space integration in the total cross section picks up the following modification factor:

$$\mathrm{d}\sigma^{\mathrm{MLS}} = \mathrm{d}\sigma^{\mathrm{ADD}} \prod_n \frac{E_n}{\omega_n} \prod_\nu \frac{\partial k_\nu}{\partial p_\nu}\bigg|_{p_i = p_f}, \qquad (10)$$

where n runs over the four initial and final states in a $2 \to 2$ process, and p_i and p_f are the total initial and final four momenta in a $2 \to 2$ process. We work out this modification factor for the process we are studying viz. $P_1P_2 \to P_3G_{\mathbf{n}}$. Note that, while the P_i are massless, the $G_{\mathbf{n}}$ are massive. Using (2) and (5), setting $\gamma = 1$ and $l_{\mathrm{Pl}}M_{\mathrm{S}} = 1$, we can easily show that

$$\prod_n \frac{E_n}{\omega_n} = \frac{s p_{\mathrm{T}} m_{\mathrm{T}}}{4M_{\mathrm{S}}^4} \coth\left\{\frac{p_{\mathrm{T}} \cosh(y_1)}{M_{\mathrm{S}}}\right\}$$
$$\times \coth\left\{\frac{m_{\mathrm{T}} \cosh(y_2)}{M_{\mathrm{S}}}\right\}$$
$$\times \prod_{i=1,2} x_i \cosh(y_i) \coth\left(\frac{x_i\sqrt{s}}{2M_{\mathrm{S}}}\right),$$
$$\prod_\nu \frac{\partial k_\nu}{\partial p_\nu} = \operatorname{sech}^2\left\{\frac{(x_1+x_2)\sqrt{s}}{2M_{\mathrm{S}}}\right\} \operatorname{sech}^2\left\{\frac{(x_1-x_2)\sqrt{s}}{2M_{\mathrm{S}}}\right\}.$$
$$(11)$$

Above, x_1 and x_2 are the momentum fractions of the hadrons carried by the initial state partons, y_1 and y_2 are the rapidities of the final state parton and the graviton, p_{T} is the transverse momentum of the final state parton, and $m_{\mathrm{T}} = \sqrt{p_{\mathrm{T}}^2 + m_{\mathbf{n}}^2}$ is the transverse mass of the graviton $G_{\mathbf{n}}$. It is instructive to check that in the decoupling limit $M_{\mathrm{S}} \gg \sqrt{s}$, the phase space correction factor goes to unity.

The sech-square modulation in (9) together with the phase space modifications in (10) and (11) constitute the complete correction. If we keep γ and $\delta = 1/(l_{\mathrm{Pl}}M_{\mathrm{S}})$ as free parameters the analytic expressions of the modifications in (9) and (11) will look more complicated, and we do not display them. Instead, we have numerically demonstrated the sensitivity of cross section due to variations of γ and δ in the next section.

3 Results

We have numerically computed the cross section for the production of the graviton KK tower in association with a jet at the Run II of Tevatron ($\sqrt{s} = 1.96$ TeV) and at the LHC ($\sqrt{s} = 14$ TeV) in both the ADD model and in the MLS scenario. There is an irreducible SM background to the signal coming from $Z + \mathrm{jet}$ production followed by the invisible decay of the Z into neutrinos. We have computed the SM background and the signal in the ADD and the MLS cases using the MRST 2001 LO parton distributions [16] (the MLS modification of the parton densities is numerically insignificant). Since the signal peaks at larger missing-E_{T} as compared to the background, the purity of the sample can be improved by employing a strong cut on the missing-E_{T}. We require the missing-E_{T} to be greater than 80 GeV for the Tevatron Run II analysis. The cut on the jet pseudorapidity is $|\eta_{\mathrm{j}}| < 2.4$. Our results are shown for the case $d = 3$. We display the results of this computation in Fig. 1 by plotting the cross section in the ADD and the MLS scenarios as a function of M_{S}. Experimentally, the $Z + \mathrm{jet}$ cross section is not known very precisely. To have a rough estimate, we multiplied the SM cross section by the appropriate luminosity to obtain the number of events, and twice the square-root of those events (back to pb units) correspond to the 2σ spread around the SM value. Thus, no detector effect has been taken into consideration. In other words, assuming purely statistical errors on the SM cross section, we have plotted the 2σ allowed bands in the figure, which may be considered as a rough estimate of the allowed zone. As expected, the MLS cross section has a steeper fall-off with M_{S}. Consequently, the lower bound on M_{S} in the MLS scenario is diluted by about 60 GeV compared to what one obtains in the conventional ADD model at the 95% confidence level.
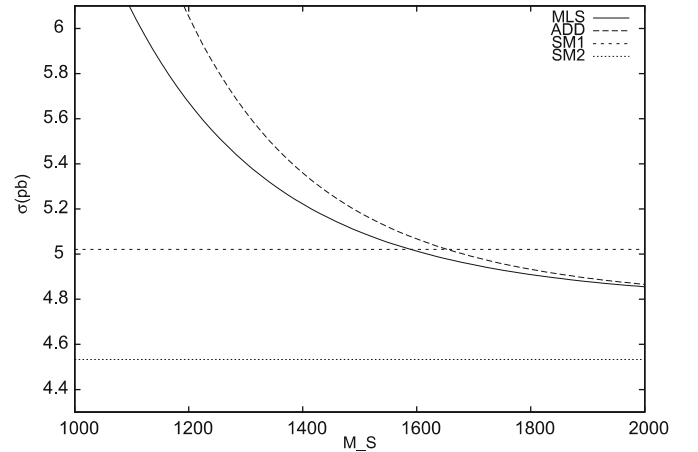

Fig. 1. The jet + missing energy cross section as a function of M_{S} for the Tevatron RUN II ($\sqrt{s} = 1.96$ TeV). The two curves are for the conventional ADD model and the MLS scenario, assuming the number of extra dimensions $d = 3$. Also shown are the 95% C.L. upper and lower bounds (SM1 and SM2, respectively) on the SM cross section (assuming only statistical errors)
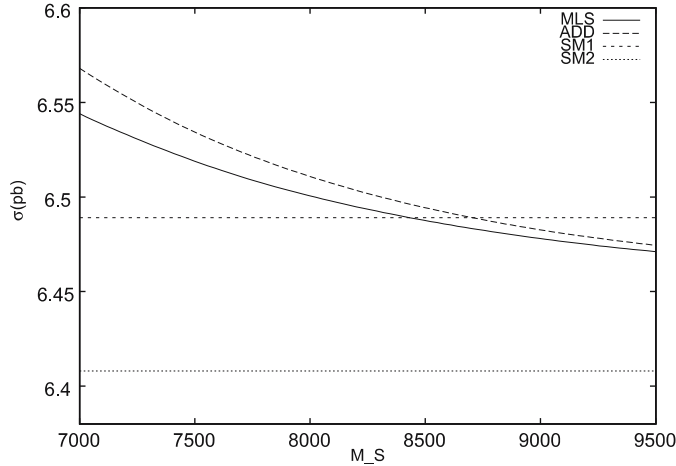
Fig. 2. Same as in Fig. 1, but for the LHC ($\sqrt{s} = 14$ TeV)
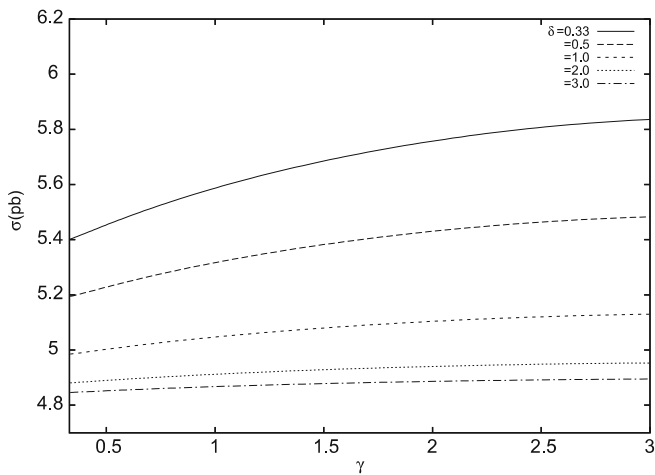
Fig. 3. Variation of the MLS cross section with γ for different values of $\delta = 1/(l_{\rm Pl}M_{\rm S})$ in the context of Tevatron RUN II ($\sqrt{s} = 1.96$ TeV). The plots correspond to the fixed $M_{\rm S} = 1.5$ TeV
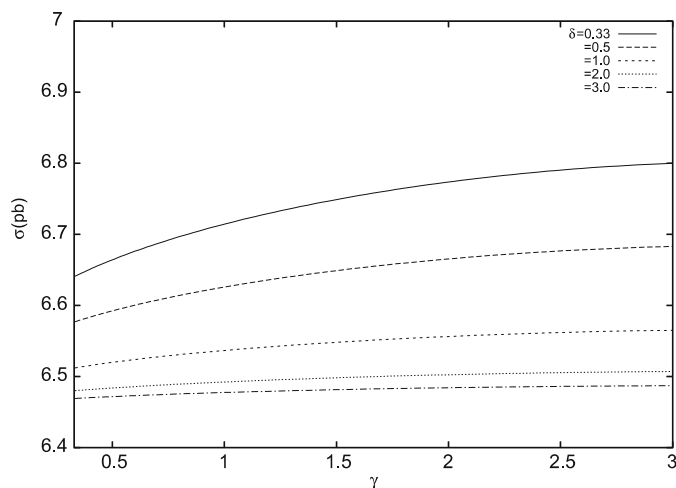
Fig. 4. Same as in Fig. 3, but for the LHC ($\sqrt{s} = 14$ TeV), with fixed $M_{\rm S} = 7$ TeV

For the LHC analysis, we have used the much stronger cut of 200 GeV on the missing $E_{\rm T}$ and a jet pseudorapidity cut of $|\eta_{\rm j}| < 3.5$. The cross section for the signal in the two cases and the SM-allowed band are shown in Fig. 2. The effects of the suppression of cross section due to the minimal length is stronger at the LHC and hence the dilution of the bound (by a few hundred GeV) is greater than at the Tevatron.

Figures 1 and 2 have been drawn setting $\gamma = 1$ and $l_{\rm Pl}M_{\rm S} = 1$. In Fig. 3, we have displayed the variation of the total cross section, for Tevatron RUN II, with γ for different choices of $\delta = 1/(l_{\rm Pl}M_{\rm S})$. The value of $M_{\rm s}$ has been fixed at 1.5 TeV for this figure. In Fig. 4, we have presented the analogous variation for the LHC energy for $M_{\rm S} = 7$ TeV. We recall that the dependence of the cross section on γ and δ arises from the use of (2) while calculating the MLS modulation factors.

4 Discussions and conclusions

The existence of a minimal length is a generic prediction of quantum gravity theories. The model of large extra dimensions, the ADD model, is an example where quantum gravity effects become manifest at energy scales as low as a TeV making it accessible to collider experiments. We incorporate the idea of a minimal length in the ADD model, which we now call the MLS scenario, and study what impact the existence of a minimal length has on the collider effects of the conventional ADD model. In particular, we have considered graviton production in association with jets at hadron colliders to address this particular issue. By analysing the jet + missing energy channel at the upgraded Tevatron and the Large Hadron collider, we demonstrate that the ADD model bounds get diluted when the minimum length hypothesis is invoked.

Finally, we make a few comments on the possible implications of a minimal length on the astrophysical constraints on the ADD model. First we note that gravitons produced in supernovaes core by a nucleon–nucleon bremsstrahlung process can carry away missing energy. But neutrino fluxes from SN1987A detected by the IMB and Kamiokande experiments have been observed to carry away most of the energy ($E \geq 2 \times 10^{53}$ ergs) released during core collapse. This has been used to obtain tight bounds on $M_{\rm S}$, of the order of 50 TeV for $d = 2$ and about 4 TeV for $d = 3$ [17] in the conventional ADD model. Since the production process of the gravitons is the same as the one studied in the present paper, it is important to ask as to what extent the MLS scenario dilutes the supernova bounds. Note that the high nucleon density at the supernova core and the large multiplicity of KK graviton modes for low values of d is responsible for the enhancement of the effects, which in turn places strong constraints on $M_{\rm S}$. The MLS scenario would indeed suppress the graviton production cross section and tend to dilute the bounds. But the numerical impact of this dilution would not be large compared to the other uncertainties, e.g. a strong core temperature dependence, inherited from pure supernova dynamics.

A string theory calculation of this cross section has been done in [18], where the amplitude is multiplied by a form factor. It is worth comparing the string theory result with the Unruh ansatz that we have used. In the string theory case the form factor goes like $\exp(-m_n^2/M_S^2)$ in the asymptotic limit. In our case, the modulation depends on the choice of γ. For $\gamma = 1$, the high mass KK states are cut off by $\exp(-m_n/M_S)$ since the argument of sech-square in (9) has a linear dependence on the graviton mass. For $\gamma = 2$, the dependence on the graviton mass is quadratic, and we obtain a string-like modulation. We note at this point that the string calculation as done in [18] is based on a toy scenario with an embedding of QED in string theory, and the calculation of jet plus missing energy relies on a rough extrapolation from this toy scenario. Because of this model dependence, a more detailed comparison of the results in [18] and our analysis is not possible.

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